四、电磁波的传播

• 1. 真空中的波动方程

Derived from Maxwell E.q.

eq $\nabla\times(\nabla\times\vec{E}) = \nabla\times\left(-\frac{\partial\vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla\times\vec{B}) = -\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$

$$\nabla\times(\nabla\times\vec{E}) = \nabla(\underbrace{\nabla\cdot\vec{E}}_{0}) - \nabla^2\vec{E} = -\nabla^2\vec{E}$$

$$\Rightarrow \begin{cases} \nabla^2\vec{E} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0 \\ \nabla^2\vec{B} - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} = 0 \end{cases}$$

samely.

where "$\nabla^2 f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$" $= 0$ ... wave equation

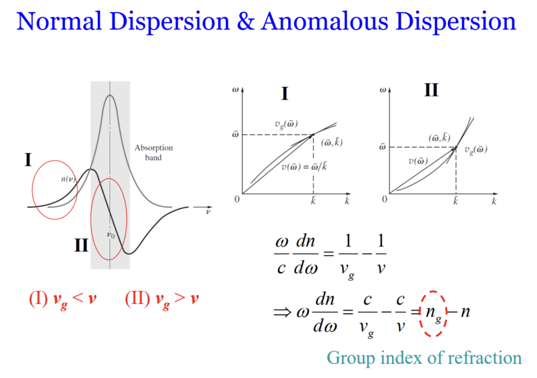
$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ = const.

Solution (plane form) $A\,e^{(\vec{k}\cdot\vec{r}-\omega t)}$

• 2. 介质的色散

∵ ε, μ are "f/ν"-dependent.

we learn in Optics:



· 5 时谐电磁波 (单色波)

(1) Def: $\vec{E}(\vec{x},t) = \vec{E}(\vec{x})\, e^{-i\omega t}$

$\vec{B}(\vec{x},t) = \vec{B}(\vec{x})\, e^{-i\omega t}$ $\qquad \vec{B} = \mu \vec{H}$.

(2) Into Maxwell. E.q.

$$\begin{cases} \nabla\cdot\vec{E} = 0 & \nabla\times\vec{E} = i\omega\mu\vec{H} \\ \nabla\cdot\vec{H} = 0 & \nabla\times\vec{H} = -i\omega\varepsilon\vec{E} \end{cases}$$

$\Downarrow$

Operator:

$\frac{\partial}{\partial t} \Leftrightarrow -i\omega$.

$\nabla\times(\nabla\times\vec{E}) = i\omega\mu\, \nabla\times\vec{H}$

$-\nabla^2 \vec{E} + \nabla(\nabla\cdot\vec{E}) = \omega^2\mu\varepsilon\, \vec{E}$

Helmholtz Eq: $\begin{cases} \nabla^2\vec{E} + k^2\vec{E} = 0 \quad (\nabla\cdot\vec{E} = 0) \\ \vec{B} = -\frac{i}{\omega}\nabla\times\vec{E} \end{cases}$

Similarly: $\nabla^2\vec{H} + k^2\vec{H} = 0$

$\vec{E} = \frac{i}{\omega\varepsilon}\nabla\times\vec{H} \qquad \vec{H} = -\frac{i}{\omega\mu}\nabla\times\vec{E}$

(3) Helmholz E.q. "$\nabla^2\vec{E} + k^2\vec{E} = 0$" where $k = \omega\sqrt{\mu\varepsilon}$.

on condition: $\nabla\cdot\vec{E} = 0$

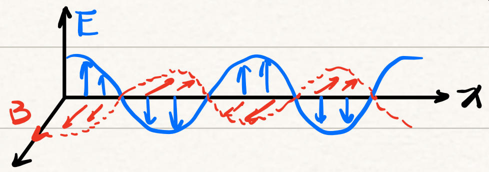
solution: $\vec{E} = \vec{E}_0\, e^{i(kx-\omega t)}$

[plane wave]

·4 时谐平面电磁波

· 振幅比 (ratio of Amplitude)

$$\frac{|\vec{E}|}{|\vec{B}|} = \begin{cases} \frac{1}{\sqrt{\varepsilon\mu}} = v & (\text{普适}) \\ \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c & (\text{cacuum}) \end{cases}$$



$$\vec{E} = \vec{E}_0(\vec{e}_x) e^{i(kx-\omega t)}$$

$k = \frac{2\pi}{\lambda}$ 角波数

· 平面电磁波 electromagnetic wave

- 横波
- $\vec{E} \perp \vec{B}$ $\therefore \perp \vec{k}$ eg $i\vec{k}\cdot\vec{E} = \nabla\cdot\vec{E} = 0$
- $\vec{E}\times\vec{B} \to \hat{k}$ 方向 $\vec{E}.\vec{B}$ 同相 $E = cB$

$$\nabla\times\vec{E} = i\omega\vec{B} = ik\times\vec{E} \Rightarrow \vec{B} = \frac{1}{\omega}k\times\vec{E}$$

$$E = v\cdot B \Leftarrow \vec{B} = \frac{1}{v}\cdot\hat{k}\times\vec{E}$$

· Energy

(1) 时谐电磁波

Energy density $w = \frac{1}{2}(\varepsilon E^2 + \frac{1}{\mu}B^2)$

E flow density $\vec{S} = \vec{E}\times\vec{H}$

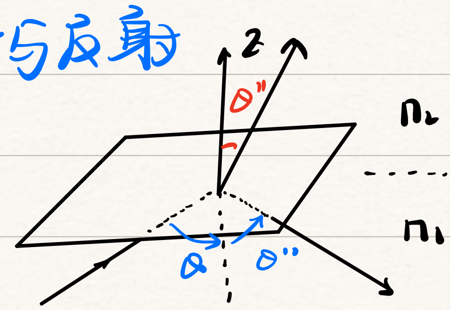
$$\bar{w} = \frac{1}{T}\int_0^T w\,dt = \frac{1}{2}\mathrm{Re}(\vec{E}^*\varepsilon\vec{E}) = \frac{1}{2}\varepsilon E^2$$

$$\bar{S} = \frac{1}{T}\int_0^T S\,dt = \frac{1}{2}\mathrm{Re}(\vec{E}^*\times\vec{H}) = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_0^2\vec{e}_k$$

(2) 全反射

$$\bar{S}_{\text{reflect}} \overset{\text{in value}}{=} \bar{S}_{\text{input}}.$$

•5. 电磁波在界面上的折射与反射



we learn in Optics:

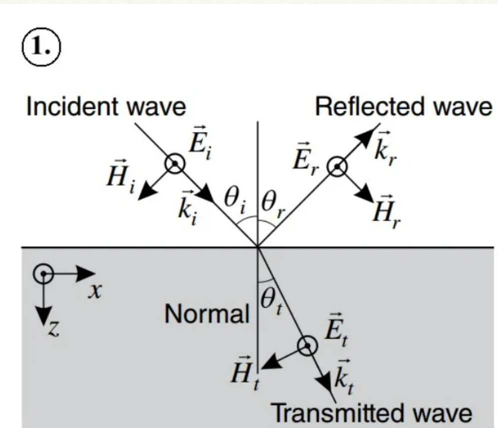
Reflect: $\theta' = \theta$

Refraction: (Snell Law)

$$\frac{\sin\theta}{\sin\theta''} = \frac{n_2}{n_1}$$

... derived from
∵ Huygen's Pricinciple

Fresnel's Law (2 types)



(1) 入射 $\vec{E}$ 垂直于纸面

$$\frac{E'}{E} = \frac{\sqrt{\varepsilon_1}\cos\theta - \sqrt{\varepsilon_2}\cos\theta''}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''} = -\frac{\sin(\theta-\theta'')}{\sin(\theta+\theta'')}$$

$$\frac{E''}{E} = \frac{2\sqrt{\varepsilon_1}\cos\theta}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''} = \frac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')}$$

(2) 入射 $\vec{E}$ 平行于纸面

$$\frac{E'}{E} = \frac{\sqrt{\varepsilon_2}\cos\theta - \sqrt{\varepsilon_1}\cos\theta''}{\sqrt{\varepsilon_2}\cos\theta + \sqrt{\varepsilon_1}\cos\theta''} = \frac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')}$$

$$\frac{E''}{E} = \frac{2\sqrt{\varepsilon_1}\cos\theta}{\sqrt{\varepsilon_2}\cos\theta + \sqrt{\varepsilon_1}\cos\theta''} = \frac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')\cos(\theta-\theta'')}$$

偏振角（Polarization angle）：当 $\theta_i + \theta_t = \frac{\pi}{2}$ 时，$r_{\parallel} = 0$, 此时的角度为 θ_p

临界角（Critical angle）光线从光密介质射向光疏介质（内反射）当入射角为某一数值时，折射角等于 $90°$，此入射角称临界角 $\theta_c = \arcsin\frac{n_t}{n_i}$。

Application.

(1) 半波损失 (电磁波垂直于界面方向入射，反射光可能出现半波损失)

- $\vec{E}$ 垂直于纸面入射：$R_{\perp} = \left(\frac{E'}{E}\right)_{\perp} = \frac{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}}$
- $\vec{E}$ 平行于纸面入射：$R_{\parallel} = \left(\frac{E'}{E}\right)_{\parallel} = \frac{\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}}$

如果 $\sqrt{\varepsilon_2} > \sqrt{\varepsilon_1}$ ，$R_{\perp} < 0$ 表示 $\vec{E}'$ 与入射方向相反，这时产生了半波损失。

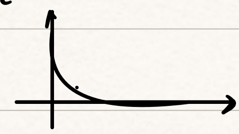
• 6. 有导体时电磁波的传播

prefare → $\rho(t)=0$ $\quad \nabla\vec{J}+\frac{\partial\rho(t)}{\partial t}=0$ & $\vec{J}=\sigma\vec{E}$

• "Conductor" $\quad$ Gauss Law $\quad \nabla\cdot\vec{E}=\frac{\rho}{\varepsilon}$

then solution to PDE → $\rho(t)=\rho_0 e^{-\frac{\sigma}{\varepsilon}t}$

$\frac{\sigma}{\varepsilon w}\gg 1.$ 良导体 $\rho(t)\to 0$ like



• Complex ε. $\quad \varepsilon'=\varepsilon+i\frac{\sigma}{w}$

不引起损耗. $\quad$ cost.

Helmholz E.q. $\quad \varepsilon'$ into Maxwell E.q.

$$\nabla^2\vec{E}+k^2\vec{E}=0, \quad \text{when } k=w\sqrt{\mu\varepsilon'}$$

$$\vec{E}(\vec{x})=\vec{E}_0 e^{ikx} \qquad \vec{k}=\vec{\beta}+i\vec{\alpha}$$

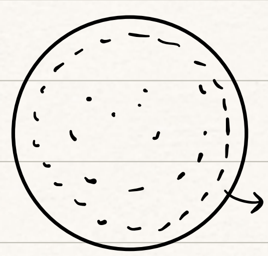
$$\Rightarrow \vec{E}=\vec{E}_0 e^{-\vec{\alpha}x}e^{i(\vec{\beta}\vec{x}-wt)}$$

α 衰减 const $\quad \beta$. 传播 (phase const)

conditions $\begin{cases}\beta^2-\alpha^2=w^2\mu\varepsilon\\ 2\vec{\beta}\cdot\vec{\alpha}=w\mu\sigma\end{cases}$

• Skin Effect.

in(交变 E/B)



$\vec{J}\to$ skin.

靠近. J大.

• Penetrate depth

E ↓ $(\frac{1}{e})$'s d.

$$\delta=\frac{1}{\alpha}=\sqrt{\frac{2}{w\mu\sigma}}$$

- 谐振腔
- 波导
- 截止频率、波长 ...

To be updated~